Step Correction of Misaligned Beam Waveguides

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This paper studies the steady-state performance of a system that stabilizes the beam position in optical waveguides. We have constrained the system to make only a single fixed amount of correction in the beam position at any given lens. We consider a symmetric corrector capable of correcting both positive and negative errors at any given lens and an unsymmetric corrector capable of correcting only positive errors at any given lens. We give the results from our studies of the performance of these systems when the lens misalignment forms a wave at the guide resonant spatial frequency, ω_0, and from our simulation of 5,000 confocal guides which were subjected to uncorrelated lens misalignment. We also derive an approximate statistical theory relating the root mean square beam displacement to the root mean square lens misalignment. We relate systems where correction occurs at every lens to systems where correction can occur only at every nth lens.

I. INTRODUCTION

Both theoretical and experimental studies of the guided transmission of optical beams indicate that some sort of active control of the beam position is required in order to keep the beam within the guide when it is transmitted over long distances.[1–7]

The optical waveguide considered here consists of a sequence of identical lenses of focal length f and spacing d, as shown in Fig. 1. The system operates by sensing the position of the beam and making discrete adjustments in the transverse positions of the optical centers of the lenses. The system maintains the beam within a given region of the guide axis. We consider steady-state performance. Various schemes that eliminate the possibility of bothersome overshoot in the transient response have been proposed.[8–10]

We consider the problem in two dimensions. It has been shown that the three-dimensional problem can be split into two separate two-

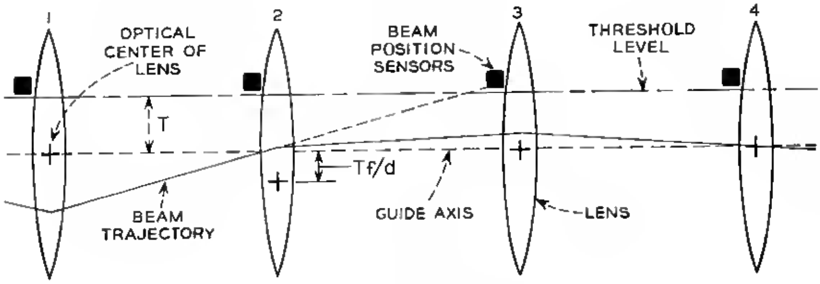

Fig. 1 — Optical waveguide.

dimensional problems.[1] Although linear systems have been shown to be capable of providing highly accurate beam position control, the desired positional stability can be obtained using simpler more efficient nonlinear mechanisms.[11] Christian, Goubau, and Mink have demonstrated experimentally and with computer simulations that a marked improvement in optical transmission can be achieved through the use of nonlinear self-aligning beam waveguides.[8]

In an effort to achieve the simplest physical design, we have constrained the system to make only a fixed amount of correction in the beam position at any given lens. When the beam displacement is large the system does not attempt to totally correct at any lens but rather accumulates correction in the same manner that lens misalignment causes beam displacement to accumulate with distance of propagation. If the beam displacement is small the system does nothing until the beam has propagated through enough misaligned lenses so that its displacement exceeds a threshold; then the beam displacement is reduced by an amount equal to the threshold. The lenses are moved in a manner that suppresses the component of the lens displacement at the guide resonant spatial frequency, $\omega_o = (1/d) \cos^{-1} (1 - d/2f)$.

The correction of the beam position at the nth lens is introduced by changing the slope of the beam at the $(n - 1)$th lens. One way to accomplish this is to induce a corrective displacement of the $(n - 1)$th lens.*

II. SYSTEMS

2.1 *The Three-State Corrector*

The three-state corrector system is capable of making either positive or negative corrections in the beam position at any lens. The

* The slope of the beam can also be changed by inserting prisms into the beam, by changing the focal length of the lenses, or by changing the distance between lenses.

amount of the correction is equal to the threshold. The name "three state corrector" arises from the number of positions available to the lenses. For this system each lens occupies either its center position or one of two alternate positions. If at the nth lens the beam displacement exceeds a threshold T, the $(n - 1)$th lens is displaced an amount $-Tf/d$ causing the beam displacement at the nth lens to be reduced by T. If the beam displacement at the nth lens passes the threshold in the negative direction the $(n - 1)$th lens would be moved an amount Tf/d to its other alternate position. If either of these corrections does not result in the beam displacement being less than the threshold no further correction can be made at this lens. However, additional correction is added at other lenses as the beam propagates.

2.2 *The Two-State Corrector*

The two-state corrector system differs from the three state system in that it is capable of making corrections in only one direction at any given lens. A lens is displaced $-Tf/d$ to its alternate position when the beam displacement at the next lens exceeds the positive threshold. Although the system is capable of correcting only positive errors at any given lens, negative errors do not become large because the beam oscillates about the guide axis as it propagates. Negative errors become positive errors after the beam has propagated a distance $d = \pi/\omega_o$.

The two-state system is capable of reducing any errors in the beam position provided the beam remains within the aperture of the guide. The correction is distributed over more lenses than with the three-state corrector.

III. WORST CASE PERFORMANCE IN A CONFOCAL GUIDE

Let us consider the worst case situation to be a sequence of equal lens displacements, D, forming a square wave at the spatial resonant frequency ω_o. The effect of each displacement adds directly to the effect of previous lens displacements. The beam displacement is proportional to the number of lenses through which it has passed. The beam oscillates about the guide axis as it propagates.

The response of the control mechanism is to generate a correction in the beam displacement which subtracts from the beam displacement caused by the lens misalignment. The increase in the beam displacement at any lens is Dd/f where D is the amount that the lens is misaligned and d/f equals two for a confocal guide.

For the three-state system the amount of correction in the beam displacement that can be introduced at any lens is equal to the

threshold level, T. In order for the beam to remain within a finite region of the guide axis the increase in the beam displacement at each lens must be less than the amount of correction that is possible, that is $D < T/2$. As long as the lens misalignment is less than $T/2$ the beam displacement can not exceed the threshold.

For the two-state system the amount of correction that can be introduced at any given lens is either T or zero, depending on whether the beam displacement is positive or negative. If, on the average, the amount of the increase in the beam displacement is to be less than the amount of the correction, D must be less than $T/4$. As long as the lens misalignment is less than $T/4$ the beam displacement can not exceed $1.5T$.

IV. RESULTS OF SIMULATIONS

Computer experiments were performed in order to evaluate the performance of the systems. The two-state and three-state systems were each simulated in 5,000 confocal guides. Each guide was subjected to a different set of uncorrelated gaussian amplitude distributed, transverse lens displacements. Quantities that were observed were: σ_r, the rms beam displacement as a function of σ_L (the rms lens misalignment), and the distribution of beam displacements at the 25th lens for various values of rms lens misalignment. All quantities are measured in units of T, the threshold displacement.

From the rms beam displacement the mean square beam displacement $\langle r^2 \rangle$ was determined and plotted in Figs. 2 and 3 as a function of the distance of propagation through the guides for the three-state and

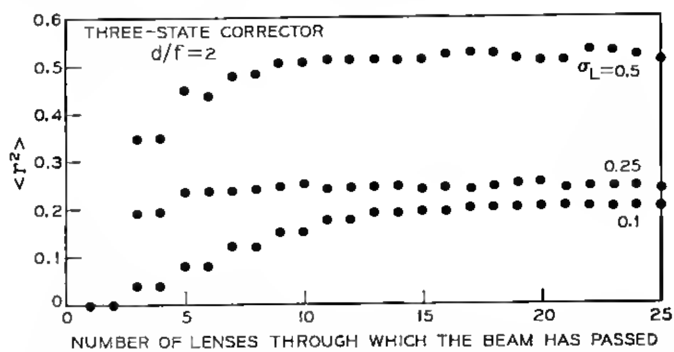

Fig. 2 — Mean square beam deviation averaged over 5,000 confocal guides with the three-state correctors at each lens.

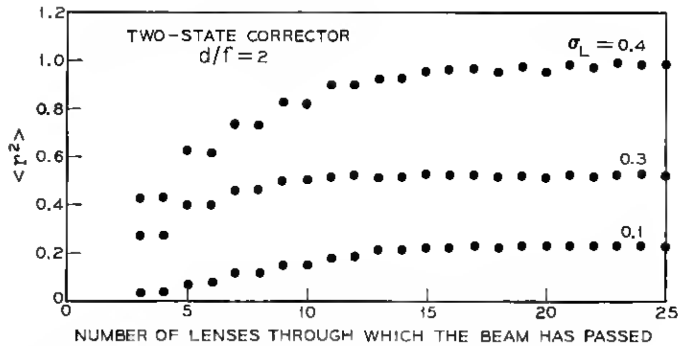

Fig. 3 — Mean square beam deviation averaged over 5,000 confocal guides with the two-state correctors.

two-state systems, respectively. The staircase appearance of the plotted points is because of the effects of the displacements of alternate lenses are independent in a confocal guide, for example, displacing even numbered lenses causes the beam displacement only at odd numbered lenses. The plots of $\langle r^2 \rangle$ in Figs. 2 and 3 approach straight line asymptotes. The increase in $\langle r^2 \rangle$ is linear until it has increased to the point where the control mechanism begins to act to maintain $\langle r^2 \rangle$ at a constant equilibrium value. In the appendix, an approximate expression relating the equilibrium value of σ_r to σ_L is derived for both the two-state and three-state position control systems. Figure 4 con-

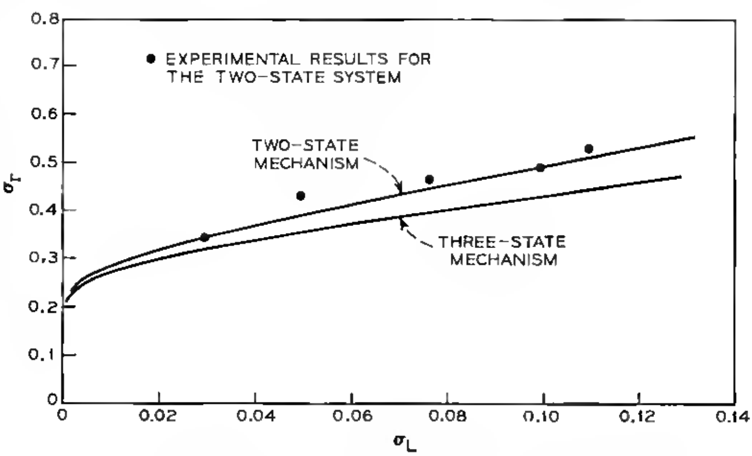

Fig. 4 — The rms beam displacement versus the rms lens misalignment.

tains plots of this approximation. The experimental points marked on the plot were obtained from computer simulations of the two-state mechanism.

Figures 5 and 6 are the distribution of beam displacements from the confocal guide axis at the 25th lens, when the guide is controlled with a three-state system. The lens displacements are uncorrelated, gaussian, and have a standard deviation, σ_L, of 0.1 in Fig. 5 and 0.5 in Fig. 6. In Figs. 5 through 9, $(\Delta n/N)/\Delta r$ is the fraction of beam displacements per unit displacement. When σ_L equaled 0.1 (Fig. 5) in the three-state system, the mechanism kept the beam displacement below the threshold in all of the 5,000 guides simulated. Figures 7, 8, and 9 are the distributions of beam displacements from the confocal guide axis at the 25th lens when the guide is stabilized using a two-state position corrector. The lack of symmetry of the corrector is evident in the distributions. A comparison of Figs. 5 and 7 shows that when the rms lens deviation is small (0.1 of the threshold) the two-state system performs nearly as well as the three-state system.

The system can be further simplified by allowing only every nth lens to be movable. Then for both the two- and three-state systems, it follows from equation (1) that the guide misalignment produces an increase in the mean square beam displacement of $2n\sigma_L^2$ between each corrector. Therefore the performance of a system where only every nth lens is adjustable is equivalent to the performance of a system where

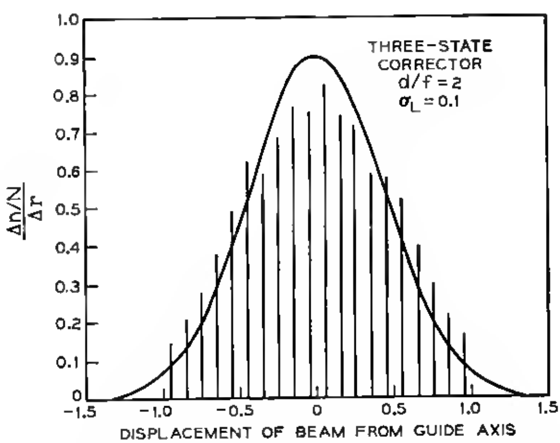

Fig. 5 — Distribution of beam displacements for a three-state corrector with the rms lens misalignment σ_L equal to 0.1.

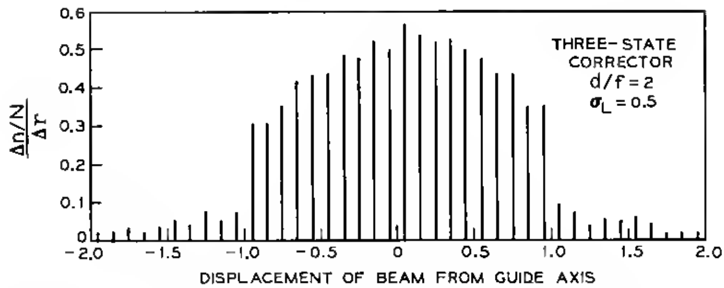

Fig. 6 — Distribution of beam displacements for a three-state corrector with the rms lens misalignment σ_L equal to 0.5.

each lens is adjustable, provided the lens displacements are uncorrelated and the mean square lens displacement is less by a factor of $1/n$.

Since, in a confocal guide, corrections in the beam position are introduced at the even lenses by displacing the odd lenses and at the odd lenses by displacing the even lenses, the distribution of correctors must be evenly spread between the even and the odd numbered lenses.

V. CONCLUSION

The use of two- and three-state beam position controllers in optical waveguides stabilizes the beam position in the guide.

When tbe misalignment of the lenses in a confocal guide forms waves at ω_o , the resonant spatial frequency of the guide, the rms displacement of the beam reaches an equilibrium value beyond which it does not

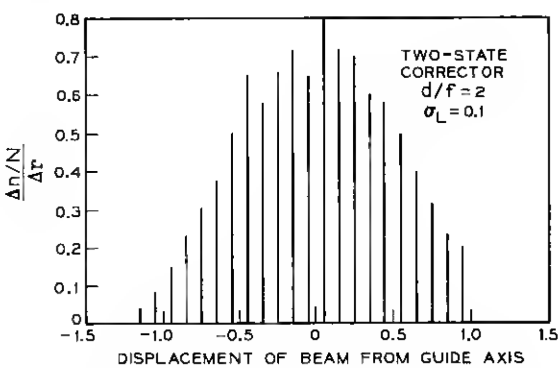

Fig. 7 — Distribution of beam displacements for a two-state corrector with the rms lens misalignment σ_L equal to 0.1.

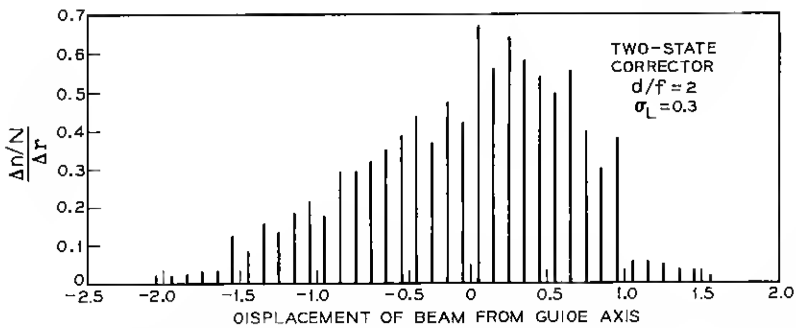

Fig. 8 — Distribution of beam displacements for a two-state corrector with the rms lens misalignment σ_L equal to 0.3.

grow, as long as the amplitude of the wave of lens displacements is less than $T/2$ for the three-state system or $T/4$ for the two-state corrector.

When the lens displacements are gaussian, uncorrelated, and have an rms value, σ_L, that is less than 0.1 of the threshold the two-state system works nearly as well as the three-state system. If $\sigma_L < 0.1$, then $\sigma_r < 0.5$ and the distribution of the beam displacements is approximately gaussian. The probability of the beam exceeding the threshold at any given lens is less than 10^{-4} if $\sigma_L < 0.1$.

APPENDIX

Approximate Statistical Theory

It has been shown that in a beam waveguide with misaligned lenses the probability distribution of the beam displacement from the guide axis is gaussian with a standard deviation that increases with increasing distance of propagation through the guide.[12] In a confocal guide the average value of this increase is given by

$$\triangle_g \sigma_r^2 = 2\sigma_L^2 \tag{1}$$

where $\triangle_g \sigma_r^2$ is the change in the mean square beam deviation resulting from guide misalignment and σ_L is the rms lens displacement.

When a discrete-state beam position control system is used, the probability distribution is stationary and the mean square beam deviation, σ_r^2 , does not increase with increasing distance of propagation. The increase in σ_r^2 resulting from lens misalignment is counteracted by a decrease in σ_r^2 resulting from the action of the controller.

Assume that $P(r)$ is the probability that beam displacement equals r before the controller acts. When the controller is a three-state cor-

rector, the mean square beam deviation after the controller acts is

$$\langle r^2 \rangle_a = 2 \int_0^1 r^2 P(r)\, dr + 2 \int_1^\infty (r - 1)^2 P(r)\, dr; \tag{2}$$

expanding the term $(r - 1)^2$ in the second integral yields the equation

$$\langle r^2 \rangle_a = \langle r^2 \rangle_b - 4 \int_1^\infty r P(r)\, dr + P(\,|\, r \,|\, > 1) \tag{3}$$

where $\langle r^2 \rangle_b$ and $P(|\, r \,| > 1)$ are the mean square beam deviation before the controller acts and the probability that the magnitude of the deviation before the controller acts is greater than one, respectively.

When $P(r)$ drops off rapidly for $r > 1$ an approximation to the integral in equation (3) can be obtained by setting r equal to one in the integral. This results in the following expression for the change in the mean square beam deviation resulting from the controller

$$\Delta_c \sigma_r^2 = \langle r^2 \rangle_a - \langle r^2 \rangle_b \approx -P(\,|\, r_b \,|\, > 1) \tag{4}$$

where $P(|\, r_b \,| > 1)$ is the probability that the magnitude of the beam deviation before the controller acts is greater than one. It is also the probability that the controller acts. Since $\Delta_g \sigma_r^2 = -\Delta_c \sigma_r^2$ it follows from equations (1) and (4) that

$$P(|\, r_b \,| > 1) \approx 2\sigma_L^2\,. \tag{5}$$

Equation (5) is the probability that the three-state system has responded to a threshold crossing. Assuming that $P(r)$ is gaussian it follows from equation (5) that

$$\sigma_L^2 = \operatorname{erf}\,(-1/\sigma_r) \tag{6}$$

where σ_r is the rms beam displacement and $\operatorname{erf}(-1/\sigma_r)$ is the error function of $-1/\sigma_r$.

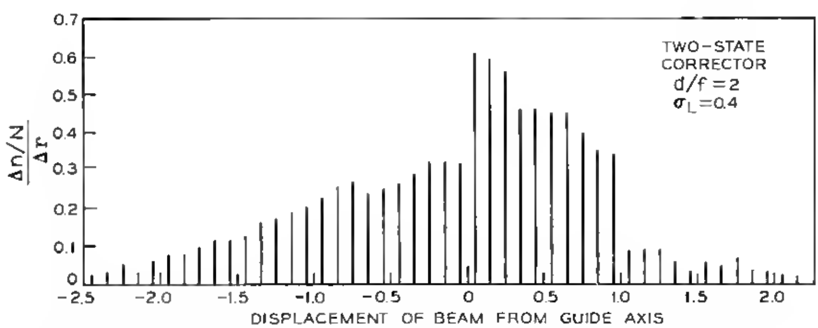

Fig. 9 — Distribution of beam displacements for a two-state corrector with the rms lens misalignment σ_L equal to 0.4.

Equation (6) is approximate. In order to apply it to the guide one must assume (*i*) that the probability distribution of beam displacements at any given lens is gaussian, and (*ii*) that the change in the distribution at any lens resulting from the guide misalignment is small. These assumptions are more accurately satisfied as the rms misalignment decreases. A comparison of the distribution obtained from the simulations when $\sigma_L = 0.1$ and the gaussian used to approximate it is shown in Fig. 5.

An analysis of the two-state system that proceeds in the same manner as above yields the following relationship between the rms beam and rms lens displacements

$$\sigma_L^2 = \tfrac{1}{2} \operatorname{erf} (-1/\sigma_r); \tag{7}$$

the probability that at any given lens the control system will respond to a threshold crossing is

$$P(r > 1) = 2\sigma_L^2 . \tag{8}$$

The relationships between σ_r and σ_L given by equations (6) and (7) are plotted in Fig. 4 along with experimental points obtained from simulations of the two-state system.

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